Finding the focal point of a dome port

by David W Knight[1]

Abstract

When a convex spherical surface (a dome) is used a barrier between a camera and a medium of high refractive index, such as water, rays from infinity appear to come from a point just in front of the dome. For water, the distance from this virtual image to the centre of curvature of the dome is about four times the dome radius. That approximation is fairly good for a thin walled dome, but the need to withstand a large pressure difference in underwater imaging practice generally demands the use of a thick-walled dome. This article gives the derivation of a more accurate formula that takes the dome thickness and refractive index into account.

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Introduction

When forming an image via a boundary between two media of widely differing refractive index, off-axis rays will deviate at the boundary. This deviation can cause image distortion; and it will, in general, also be wavelength dependent, thus causing chromatic aberration. Aberration becomes particularly serious when using a wide field-of-view (FOV). Underwater imaging is the most common situation in which this effect is encountered. The full solution to the problem is, of course, to include the boundary in the overall lens design and optimisation process; but the more usual (cost effective) approach is that of adapting an existing lens by placing it behind a hemispherical window, this structure being known as a 'dome port'.

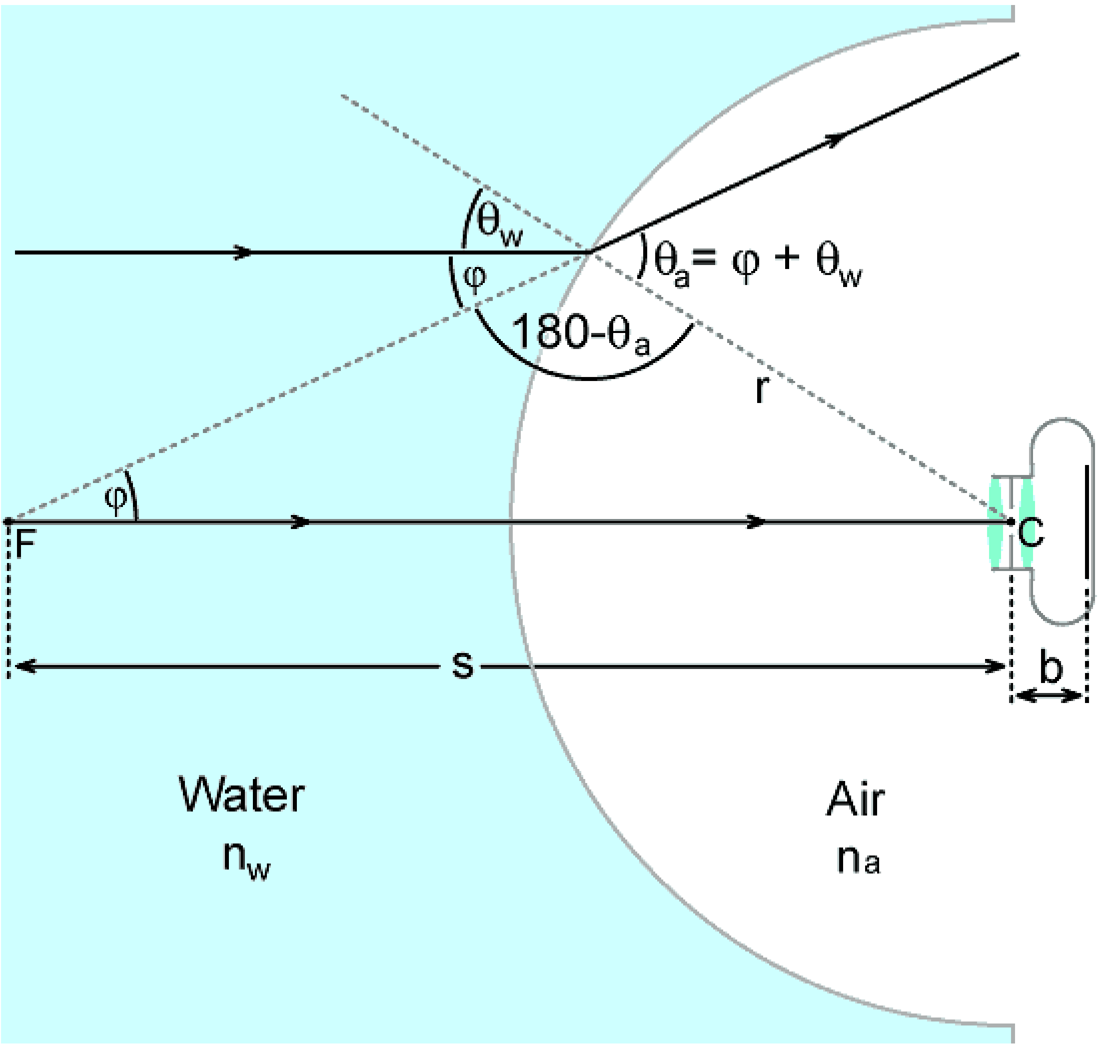
The focal point of a lens is the point at which light from an object at infinite distance is made to converge. Since an underwater dome port is a diverging lens, the focal length is strictly negative (the image is virtual), but we will treat it as a positive quantity here because the intention is to derive an expression that can be used to estimate the distance from the camera to the virtual image.

In standard underwater (UW) photographic practice with lenses of moderate to wide field-of-view (FOV), an attempt is made to place the lens entrance pupil at the centre of curvature of the dome. This choice ensures that rays normal to the dome surface are undeflected on their path to the lens, thereby making the FOV underwater the same as it is in air. Note however that pupil positioning is rarely accurate when using an interchangeable lens camera, being dependent on generally limited choices for the distance between the lens and the port, and being subject to the fact that the camera is always mounted (inside the waterproof housing) in the same place, regardless of which lens is in use. Thus, although we must assume that the lens is in the right place for the purposes of theory, it is important to be aware that incorrect positioning will introduce error in practice.

The thin dome approximation

In the following diagram, the entrance pupil of a camera lens is placed at the centre of curvature C of an air-filled thin-walled dome of radius r that is immersed in water. A ray travelling along the lens axis is undeflected and goes straight to the camera, but an off-axis ray is deflected through an angle φ. Thus rays from infinity appear to come from a point F (the secondary focus). If the camera lens is to be able to focus on the virtual image at F, it must have a minimum focusing distance of $\leq s + b$ (where b is the back-focal distance).

1 Cameras Underwater Ltd. www.camerasunderwater.co.uk



Using the Sine rule:

$s / \sin(180-\theta_a) = r / \sin\varphi$

but $\sin(180-\theta) = \sin\theta$

Therefore

$s / \sin\theta_a = r / \sin\varphi$

In the small angle limit, $\sin\theta \rightarrow \theta$, therefore

$s = r\,\theta_a / \varphi$

but, by inspection, $\varphi = \theta_a - \theta_w$, hence:

$s = r\,\theta_a / (\theta_a - \theta_w)$

Factoring θ_a from the denominator and cancelling gives:

$s = r / (1 - \theta_w / \theta_a)$ . . . . (1)

By Snell's law of refraction:

$n_w \sin\theta_w = n_a \sin\theta_a$

(where n_w and n_a are the refractive indeces of water and air). In the small angle limit this becomes:

$\theta_w / \theta_a = n_a / n_w$

substituting this into (1) gives:

$s = r / (1 - n_a / n_w)$

If we use the approximations $n_a = 1$ and $n_w = 4/3$, then $1 - n_a / n_w = 1/4$, and

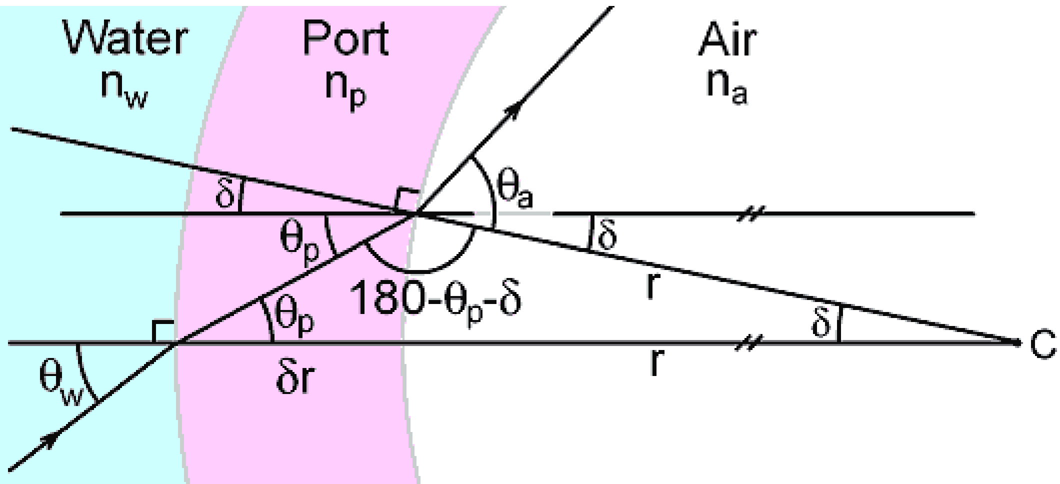
$s = 4r$

Thus, for a thin dome immersed in water, the virtual image of an object at infinity appears at a distance of about 4 radii from the centre of curvature. On the somewhat unreliable assumption that the lens pupil is correctly placed at the centre of curvature; adding 4r to the back focal distance b (which can be obtained from the lens data or estimated by noting the apparent position of the iris) gives an upper-limit estimate of the required minimum focusing distance for the camera lens.

In practice, the use of very thin domes is not possible in pressure-resistant underwater housings. If the dome has finite thickness, and is made of material having a refractive index greater than that of water (such as glass or acrylic), then the virtual image will be moved slightly closer to the camera. Thus the minimum focusing requirement is a little more severe than the simple 4r rule would seem to imply.

Thick-walled dome-port formula

The derivation of the focal distance for the general case is somewhat more tricky than the thin-wall approximation, but we can start by analysing the passage of a light ray striking the port at an arbitrary angle.



In the diagram above, an incident ray (in water) meets the port surface at an angle θ_w to the perpendicular and emerges into the port material at an angle θ_p. The relationship between the two angles is given by Snell's law in the small-angle limit:

$$n_w \theta_w = n_p \theta_p \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

As the ray traverses the port to strike the inner surface, the normal to the surface undergoes an angular displacement δ . Hence the angle of incidence at the inner surface is $\theta_p + \delta$. If the ray exits the port material into the air at an angle θ_a , then Snell's law gives:

$$n_p (\theta_p + \delta) = n_a \theta_a \qquad \ldots\ldots\ldots (3)$$

By inspection of the diagram; using the Sine rule we get:

$$\mathrm{Sin}(180 - \theta_p - \delta) / (r + \delta r) = \mathrm{Sin}\theta_p / r$$

Where r is the dome inner radius, and δr is the dome thickness. Now, since $\mathrm{Sin}(180-\theta) = \mathrm{Sin}\theta$, in the small angle limit this becomes:

$$(\theta_p + \delta) / (r + \delta r) = \theta_p / r$$

Rearrangement gives:

$$\delta = \theta_p \delta r / r \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

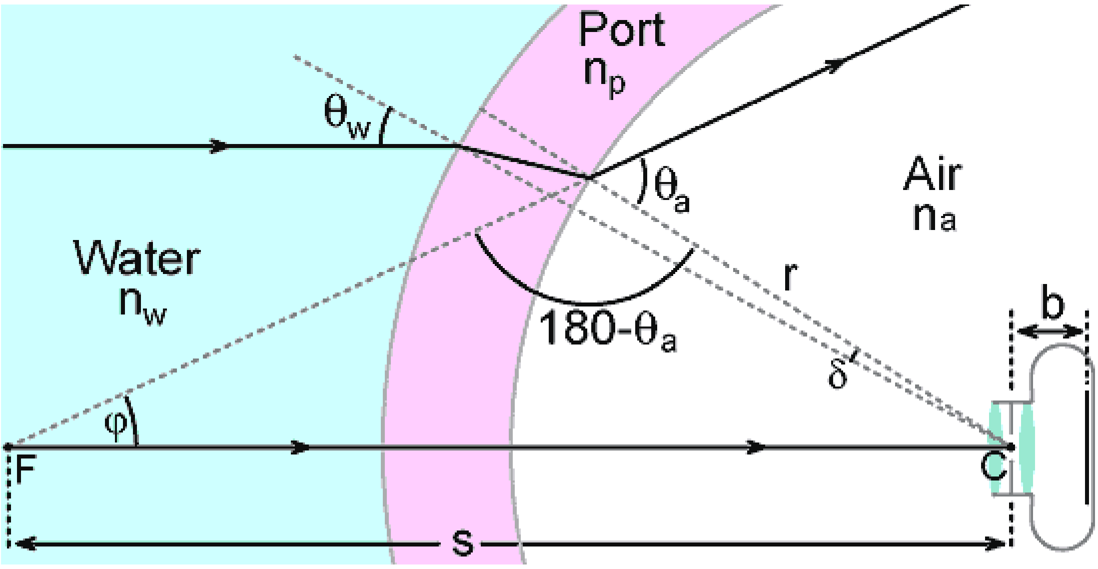
Substituting this into (3) gives:

$$n_p \theta_p (1 + \delta r / r) = n_a \theta_a$$

and using (2) to substitute for θ_p gives:

$$\theta_w / \theta_a = (n_a / n_w) / (1 + \delta r / r) \qquad \ldots\ldots\ldots (5)$$

As shown below, an expression for the focal distance s is obtained in the same way as for the thin-walled case:



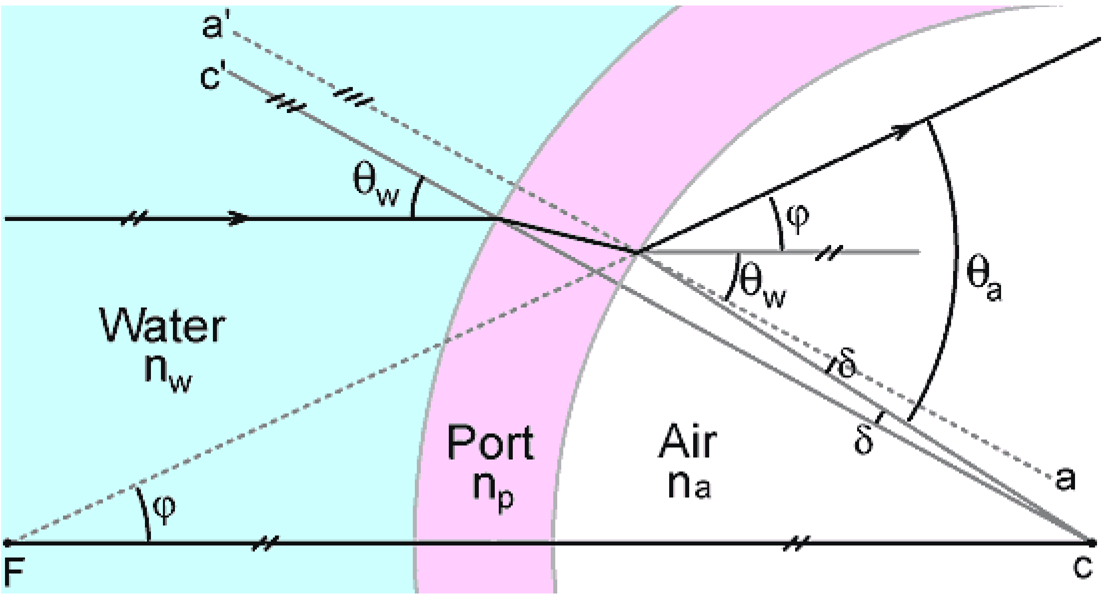
Referring to the diagram above, using the Sine rule gives:

$s / \sin(180 - \theta_a) = r / \sin\varphi$

but $\sin(180 - \theta) = \sin\theta$, and so, in the small angle limit:

$s / \theta_a = r / \varphi$ . . . . (6)

A problem that remains however, is that of eliminating φ so that s can be expressed in terms of fixed physical parameters. The solution is obtained by drawing parallels, as shown in the diagram below:



by drawing a line a - a' parallel to the line c - c' against which θ_w is defined, we can see that:

$\theta_a = \varphi + \theta_w + \delta$

i.e.,

$\varphi = \theta_a - \theta_w - \delta$

Using this in (6) gives:

$s = r\,\theta_a / (\theta_a - \theta_w - \delta)$

Substituting for δ using (4) gives:

$s = r\,\theta_a / (\theta_a - \theta_w - \theta_p\,\delta r / r)$

and substituting for θ_p using (2) gives:

$s = r\,\theta_a / [\ \theta_a - \theta_w - \theta_w\,(n_w / n_p)\,\delta r / r\ ]$

i.e.,

$s = r\,\theta_a / [\ \theta_a - \theta_w\ \{ 1 + (n_w / n_p)\,\delta r / r \}\ ]$

Factoring θ_a from the denoninator and cancelling gives:

$s = r / [\ 1 - (\theta_w / \theta_a)\ \{ 1 + (n_w / n_p)\,\delta r / r \}\ ]$

and substituting for θ_w / θ_a using (5) puts the focal distance in terms of fixed system parameters and solves the problem:

$s = r / [1 - (n_a / n_w)\ \{1 + (n_w / n_p)\,\delta r / r\} / (1 + \delta r / r)\ ]$

Example calculations

Some example calculations are shown below[2]. Accurate data were available for the ikelite 6" port, but the thicknesses of the boron crown-glass (BK7) ports are guesses. Still, the calculations give the general idea, which is that the finite thickness of the port puts the virtual image some 10 to 15mm closer to the camera than the simple 4r rule would suggest.

n_a	1.00028
n_w	1.339
n_a / n_w	0.74704

$$s = \frac{r}{1 - \frac{n_a}{n_w}\left(1 + \frac{n_w}{n_p}\frac{\delta r}{r}\right) \Big/ \left(1 + \frac{\delta r}{r}\right)}$$

	radius	thickness	refr idx				foc dist	∞ corr
Dome port	r /mm	δr /mm	n_p	s_{approx} = 4r	n_w / n_p	δr / r	s / mm	Diopters
ikelite 6" acrylic	76.2	5.537	1.492	304.8	0.8975	0.0727	295.2	3.39
4" BK7	50	6	1.5168	200	0.8828	0.1200	190.6	5.25
8" BK7	100	6	1.5168	400	0.8828	0.0600	387.7	2.58
9" BK7	115	7	1.5168	460	0.8828	0.0609	445.8	2.24
9.25" BK7	117.5	7	1.5168	470	0.8828	0.0596	455.6	2.19

2 This is a snapshot of an open document spreadsheet ' dp_calcs.ods ', which can be downloaded along with this document. The spreadsheet can, of course, be used as a template for other calculations. To access the file, It is necessary to install Open Office (www.openoffice.org).

The infinity correction figure, in diopters, is simply the reciprocal of s in metres, i.e., it is the magnifying power required to make rays from the virtual image become parallel if the correcting lens is placed at the centre of curvature of the dome.

Note that it is necessary to add the back focal distance b when estimating the minimum focusing distance requirement. The position of the focal plane in an interchangeable lens camera is often marked with the symbol ⦵ . The position of the lens entrance pupil should be given in the data sheet, but it can also be estimated by setting a small aperture and noting the apparent position of the iris.

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An alternative derivation is given on the 'scubageek' diving physics website[3].

For general reference, see: Jenkins and White, 'Fundamentals of Optics' [4]

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3 main index: scubageek.com
Secondary focal point of a dome port: scubageek.com/articles/wwwprim.html
Optics of dome ports: scubageek.com/articles/wwwdome.html

4 ISBN 0-07-256191-2
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